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# Sphere drag coefficient for subsonic speeds in continuum and free-molecule flows

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An extensive series of measurements of sphere drag coefficients has been made in an aeroballistic range for a broad range of Reynolds and Mach numbers. These measurements have been compared with those obtained in other test facilities. As a result of this comparison it has been possible to suggest reasons for many of the inconsistencies in the earlier measurements and to establish more accurate values of the sphere drag coefficient for $M_\infty \lesssim 0{\cdot}2$ and $10^{-2} \lesssim Re_\infty \lesssim 10^7$.

## 1. Introduction

The measurement of sphere drag coefficients at low speeds has been the subject of numerous experimental studies for at least 250 years, cf. Newton (1719). Many diverse techniques have been used to make these measurements, e.g. (i) freely falling spheres in a liquid, (ii) freely falling spheres in air, (iii) atmospheric and variable-density wind tunnels, (iv) towed spheres in water and air, (v) a sting-mounted sphere on an aircraft and (vi) the aeroballistic range.

Many analyses and reviews of these data have been made in the past and a 'standard drag' curve has been derived for a sphere travelling at subsonic speeds, cf. Hoerner (1958, pp. 3–8). In reviewing the data for $Re_\infty > 10^2$ it was not clear why some of the data were given more credibility than others when the standard curve was derived. Some comparatively recent measurements in wind tunnels, free-fall facilities and aeroballistic ranges have been evaluated with a view to determining the credibility of some of these measurements. It should be stated at the outset of this discussion that no attempt has been made to discuss all of the measurements of subsonic sphere drag that have been made. The purpose of this paper is to identify some of the experimental factors that affect the value of the sphere drag coefficient and suggest, if necessary, modifications to the generally accepted 'standard drag' curve.

## 2. Sphere drag coefficient for $10^{-2} \lesssim Re_\infty \lesssim 10^3$

For this Reynolds number range, the bulk of the values of $C_D$ have been derived from the determination of the terminal velocity of spheres as they fall through a variety of liquids. Examples of the data obtained using this technique by Arnold (1911), Liebster (1927) and Schmiedel (1928) are shown in figure 1. More recently, Maxworthy (1965) obtained some measurements, using the free-

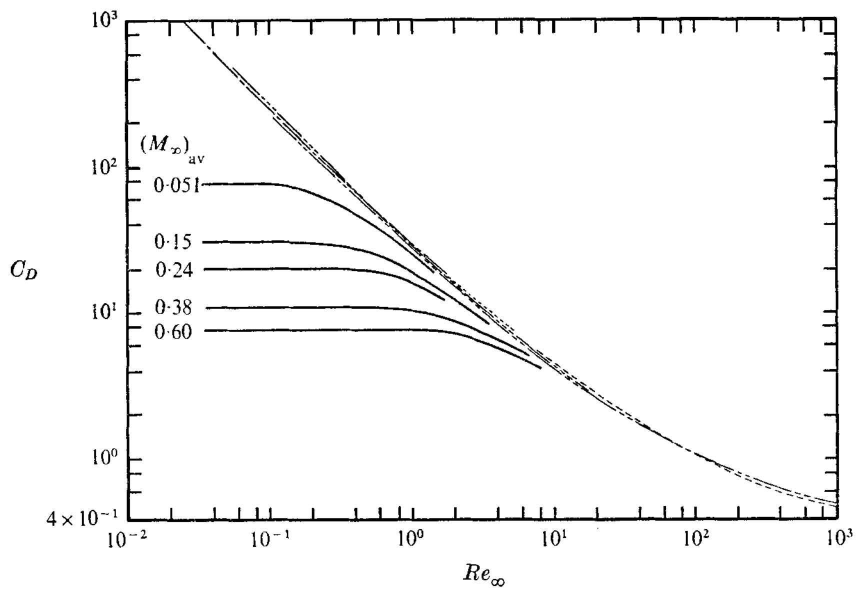


FIGURE 1. Sphere drag coefficient at low Reynolds numbers. ——, Crowe *et al.* (1971); — — —; Roos & Willmarth (1971); —·—, Arnold (1911); —··—, Liebster (1927); —···—, Schmiedel (1928); ······, Maxworthy (1965).

fall technique, that are in excellent agreement with the earlier data, cf. figure 1. Roos & Willmarth (1971) have obtained values of sphere drag by towing a sting-mounted sphere through water which are in good agreement with those of Liebster (1927). There is good agreement (within $\pm 5\%$) between all these measurements obtained for 'creeping flow' or 'Stokes flow'.

Recently Crowe, Babcock & Willoughby (1971) have measured $C_D$ at low speeds and Reynolds numbers by studying the motion of small spheres in a microballistic range. A summary of their low speed data is shown in figure 1. These data emphasize the difference between the values of $C_D$ obtained for 'Stokes flow' and rarefied flow conditions.

## 3. Aeroballistic-range measurements

The present sphere drag investigation was undertaken in the Hyperballistic Range of the von Kármán Gas Dynamics Facility of the Arnold Engineering Development Center. This range is a variable-density free-flight test unit that can be used for either aerophysical testing or classical aerodynamic tests. From a consideration of the equation of motion of a sphere in free flight, the sphere drag coefficient can be shown to have the following form:

$$C_D = \frac{8mRT}{\pi p d^2 v}\frac{dv}{dx},$$

where $m$ = mass, $v$ = velocity, $C_D$ = drag coefficient, $R$ = gas constant, $T$ = ambient temperature, $p$ = ambient pressure and $x$ = longitudinal distance. The

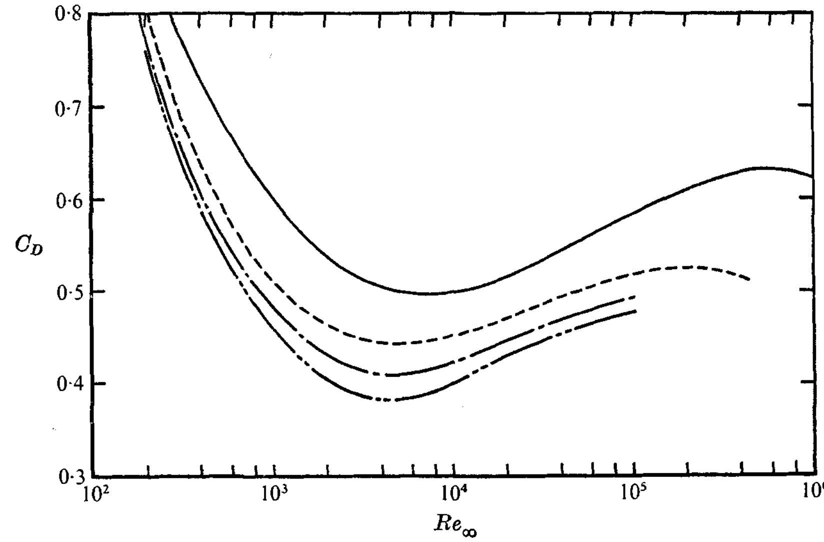


FIGURE 2. Sphere drag coefficient as a function of Mach and Reynolds numbers: Bailey & Hiatt (1971), aeroballistic range. ——, $M_\infty = 0{\cdot}8$; – – –, $M_\infty = 0{\cdot}6$; —·—, $M_\infty = 0{\cdot}4$; —··—, $M_\infty = 0{\cdot}15$.

maximum probable uncertainties in the drag coefficient attributable to uncertainties in measurements of the above parameters have been found to be 0·02 % for $v$, 0·2 % for $T$, 1·0 % for $dv/dx$, 0·1–0·3 % for $m$, 0·2–0·3 % for $d$ and 0·5 % for $p$. From a consideration of these uncertainties and the overall consistency of the data it has been concluded that total errors in the sphere drag coefficients are no greater than ± 2 %.

A smooth-bore cold gas gun was used to accelerate spheres of solid copper, beryllium–copper, aluminium and nylon to the required velocities. These spheres were enclosed in a plastic sabot to protect their surface from damage during launching. The acceleration forces acting on the model during launching were insufficient to produce any distortion. Experience has indicated that no detectable spin is imparted to the model in this type of launching process.

A complete description of the range, together with an evaluation of the accuracy of the measured distance, time, velocity, deceleration, mass, diameter, temperature, pressure, drag coefficient, Reynolds number and Mach number and a complete list of the experimental measurements are given by Bailey & Hiatt (1971). A summary of the $C_D$ values obtained at subsonic speeds is shown in figure 2.

## 4. Sphere drag coefficient for $Re_\infty > 10^2$

At low speeds and for Reynolds numbers greater than 100 the flow field around a sphere undergoes changes which determine the variation of $C_D$ with $Re_\infty$. The drag on a sphere can be considered to be comprised of two parts: (i) skin-friction drag and (ii) the drag caused by the formation of the wake. For $Re_\infty < 10^2$ the wake behind a sphere is laminar, extremely narrow and does not contribute a significant portion of the total drag on the sphere. This laminar wake appears to be very stable for these conditions since some sting-mounted measurements of $C_D$ obtained by Roos & Willmarth (1971) are in good agreement with Liebster's (1927) free-fall measurements (cf. figure 1). For $Re_\infty > 10^2$ a series of regular vortex patterns is formed in the wake of the sphere and with increasing Reynolds number becomes irregular and turbulent in character. Finally, when $Re_\infty > 10^5$ (supercritical regime) the flow on the sphere changes from laminar to turbulent and the point of separation, which lies near the equator for laminar flow, moves downstream. This results in a considerable decrease in the dead-air region behind the sphere which in turn results in a significant sudden reduction in pressure drag. If the sphere flow field is modified as a result of the particular technique being used to obtain the sphere drag the resulting value of $C_D$ is likely to be in error. In the subsequent discussion attempts will be made to determine what effect a particular experimental technique has upon the measured value of $C_D$.

Ideally, to make an analysis of this type it is necessary to know the variation of the absolute value of $C_D$ with $Re_\infty$. Since this knowledge is not available at this time the standard for comparison will be the aeroballistic-range data of Bailey & Hiatt (1971). The basic data upon which the summary curves shown in figure 2 have been based have the following characteristics: (i) they are accurate to better than $\pm 2\,\%$, (ii) they are internally consistent and show consistent trends with Mach and Reynolds number, (iii) they are free-flight support-free measurements and (iv) the tests were made in a well-controlled non-turbulent environment.

Recently, Hill & Zukoski (1972) measured the drag on spheres falling through a column of liquid. The vessel containing the liquid was attached to a vibrator which could apply oscillations of various frequencies and amplitudes to the liquid column. They found that, for $Re_\infty = 3000$, when the amplitude of the oscillation was $2\,\%$ of the diameter of the sphere a frequency of oscillation could be found that increased $C_D$ by $25\,\%$. The frequency of this oscillation was approximately the Strouhal frequency. As a result of this observation they suggested that there is a nonlinear interaction between wake vortex shedding and the oscillation in translational motion. For $Re_\infty \lesssim 300$, where the wake is attached to the sphere, the wake is stable and is not significantly affected by translational oscillations.

Brush, Fox & Ho (1969) have measured the fall velocities of spherical particles settling in water in a container that could be oscillated horizontally or vertically. They found that the fall velocity through the liquid decreased when the liquid was oscillated. This implies an increase in $C_D$ similar to that observed by Hill & Zukoski (1972).

Sivier (1967) and Zarin (1969) have made support-free measurements of sphere drag with a magnetically suspended sphere in a vertical wind tunnel. Zarin (1969) indicates that the model positioning system used by Sivier (1967) permitted large amplitude (with respect to the size of the model) oscillations of the sphere in the vertical direction. By modifying the positioning device he was able to reduce the amplitude of this oscillation significantly, which resulted in a reduction in magnitude of the sphere drag. These measurements are compared with the aeroballistic-range data in figure 3. For $Re_\infty \lesssim 8\times 10^2$ the two sets

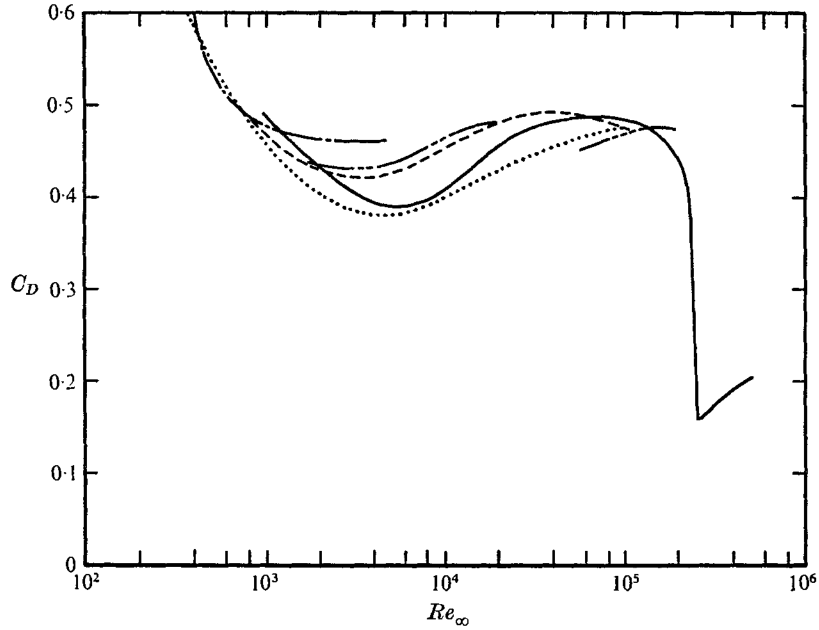


FIGURE 3. Sphere drag coefficient as a function of Reynolds number: comparison of aeroballistic-range data with wind-tunnel values. ——, Wieselsberger (1922); – – –, Roos & Willmarth (1971); —·—, Zarin (1969); —··—, Heinrich *et al.* (1963); —···—, Vlajinac & Covert (1972); ······, Bailey & Hiatt (1971).

of data are on good agreement. This good agreement is consistent with Hill & Zukoski's (1972) observations cited above, i.e. when the wake is laminar translational motion of the model will not have a significant effect upon sphere drag. Hill & Zukoski (1972) indicate that in their experiment model oscillations would be expected to have an effect for $Re_\infty > 300$, whereas it is suggested above that Zarin's (1969) measurements do not indicate an effect until $Re_\infty > 800$. Zarin's (1969) data were obtained for $M_\infty \approx 0{\cdot}1$ whereas Hill & Zukoski's (1972) were obtained for $M_\infty < 0{\cdot}1$. The Reynolds number at which transition occurs in the wake of a sphere increases with increasing speed, which provides an explanation for the difference between these two sets of measurements.

Roos & Willmarth (1971) made measurements of $C_D$ with their sting-mounted sphere towed through water up to $Re_\infty \approx 10^5$. For $Re_\infty > 5 \times 10^3$ they observed oscillation of the model and considerable scatter in the measurements, i.e. $\pm 8\,\%$. These data are compared with the aeroballistic-range data in figure 3. For $Re_\infty \lesssim 10^3$ the agreement is good whereas for $Re_\infty > 10^3$ their values are greater than the aeroballistic-range values. Thus, it seems reasonable to conclude that both Zarin's (1969) and Roos & Willmarth's (1971) measurements are affected by translational oscillations of the sphere.

The values of $C_D$ obtained by Heinrich, Niccum & Mark (1963) for

$$2 \times 10^3 \lesssim Re_\infty \lesssim 2 \times 10^4$$

in a wind tunnel are shown in figure 3. These measurements are in good agreement with those of Roos & Willmarth (1971), cf. figure 3. It is suggested that the

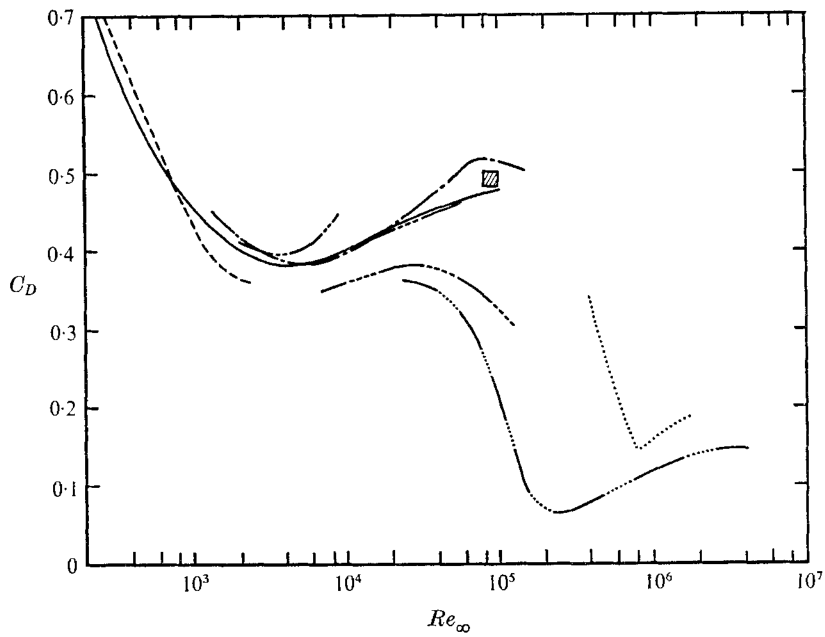


FIGURE 4. Sphere drag coefficient as a function of Reynolds number: comparison of aeroballistic-range data with free-fall values. ▨, Newton (1719); ——, Bailey & Hiatt (1971); – – –, Liebster (1927); —·—, Lunnon (1928); —··—, Allen (1900); —···—, Shakespear (1914); —····—, Lunnon (1926); —······—, ······, Bacon & Reid (1924).

wire suspension system used by Heinrich *et al.* (1963) could have permitted some translational oscillation of the model which would explain their high values of $C_D$.

Wieselsberger (1922), using a different wire suspension and balance configuration, obtained values of $C_D$ which are in reasonable agreement with the aeroballistic-range data for $10^3 \lesssim Re_\infty \lesssim 10^5$, cf. figure 3.

Lunnon (1926) made an extensive series of sphere drag measurements by timing the fall of spheres in a mineshaft. His values of $C_D$, shown in figure 4, are significantly lower than the values obtained in the aeroballistic range. A re-examination of these values by Lunnon (1928) led him to the conclusion that his measurements had been affected by flow turbulence in the mineshafts.

It has been noted earlier that for $Re_\infty \approx 10^5$ the flow over a sphere changes from laminar to turbulent and there is sudden large decrease in the drag when this occurs. In many of the experimental studies of sphere drag attempts were made to associate the Reynolds number at which the sphere drag decreased with the degree of turbulence in the flow. With increasing flow turbulence the Reynolds number at which the drag jump occurred decreased. Some $C_D$ measurements for spheres in an early version of the NACA variable-density tunnel (Bacon & Reid 1924), where the flow was known to be highly turbulent, are shown in figure 4. These wind-tunnel values and Lunnon's (1926) mineshaft data are in reasonable agreement, which suggests that Lunnon's hypothesis that his drag measurements were affected by flow turbulence is correct.

Shakespear (1914) timed the fall of a series of celluloid spheres and the derived values of $C_D$ shown in figure 4 are in good agreement with the aeroballistic-range data.

Newton (1719) timed the fall of a series of glass spheres and inflated pigs' bladders, from which values of $C_D$ have been derived; see figure 4. These values are in reasonable agreement with both Shakespear's (1914) and the aeroballistic-range values. Newton also made an extensive study of the fall of spheres through water columns. In both these studies he found that the sphere trajectory was nonlinear. If the trajectory were highly nonlinear, then a technique for determining velocity by measuring the time taken to fall through a known vertical distance would result in a velocity that was too low. This would in turn result in the derivation of too high a $C_D$ value.

Lunnon (1928) observed that for heavy spheres falling through water "there is always some swerving in the path of falling spheres". He postulated that this swerving motion results from the periodicity of the wake vortex phenomenon. The results of Lunnon's (1928) experiments are shown in figure 4. For

$$1{\cdot}5 \times 10^3 \lesssim Re_\infty \lesssim 2{\cdot}5 \times 10^4;$$

these $C_D$ values are in good agreement with the aeroballistic-range data. For $Re_\infty > 5 \times 10^5$ these values are significantly larger than the aeroballistic-range values. It is possible that these high values of $C_D$ result from an underestimate of the terminal velocity because of the nonlinear trajectory of the sphere. Allen (1900) made some free-fall measurements for steel spheres in water for

$$2{\cdot}3 \times 10^3 \lesssim Re_\infty \lesssim 9 \times 10^3,$$

see figure 4. A characteristic of these measurements is the abrupt increase in $C_D$ with increasing $Re_\infty$ for $Re_\infty > 5 \times 10^3$, which is similar to that which Lunnon (1926) observed for $Re_\infty \gtrsim 5 \times 10^5$. The possibility also exists that this $C_D$ increase results from a nonlinear model trajectory. However, a comparison of Lunnon's and Allen's results suggests that in the upper Reynolds number range of both investigations the vertical fall distance was insufficient for the terminal velocity to be achieved.

Vlajinac & Covert (1972) have measured the drag on spheres using a support-free magnetic model suspension technique in a wind tunnel. The results of these measurements are in good agreement with the aeroballistic-range data; see figure 3.

In the supercritical regime there are considerable differences in the absolute values of $C_D$ depending upon the technique used to make the measurements. Some measurements obtained by Bacon & Reid (1924), Wieselsberger (1922), Flachsbart (1927), Millikan & Klein (1933), Hoerner (1958) and Achenbach (1972) are compared in figure 5. Millikan & Klein, Hoerner and Achenbach obtained values of $C_D$ using a sting-mounted sphere, on an aeroplane, towed in free air and in a conventional wind tunnel respectively. It is evident from this comparison that, for a sting-mounted model, $C_D$ is a function of the manner in which flow over the sphere is generated. Flachsbart (1927) modified Weiselsberger's (1922) wire mounting system and affected the resulting value of $C_D$; see figure 5.

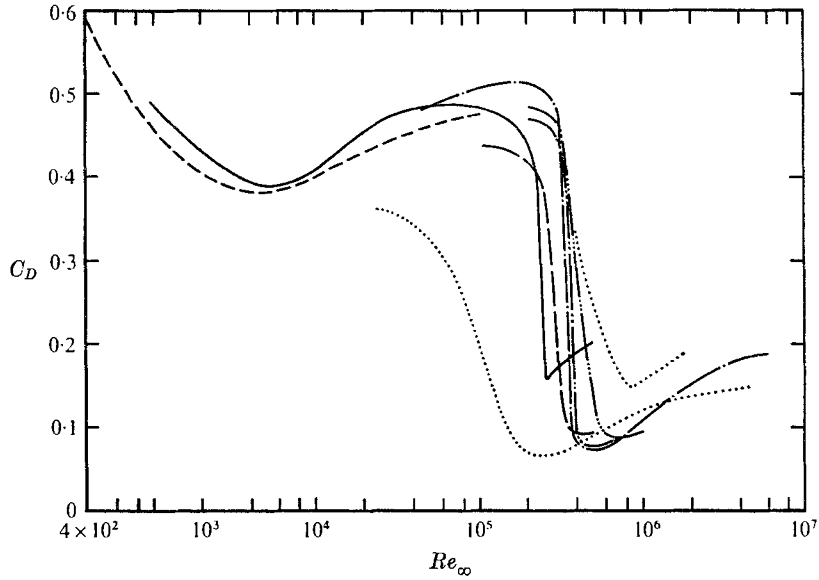


FIGURE 5. Sphere drag coefficient as a function of Reynolds number: supercritical flow. ——, Wieselsberger (1922); – – –, Bailey & Hiatt (1971); — — —, Flachsbart (1927); —·—, Achenbach (1972); —··—, Millikan & Klein (1933); —···—, Hoerner (1958); ······, Bacon & Reid (1924).

The $C_D$ values obtained with the wire suspension system differ from the sting-mounted data discussed above, indicating that the value of $C_D$ is a function of the type of model suspension system. The effect of wind turbulence on a wire-supported model is well illustrated by Bacon & Reid's (1924) measurements in the NACA variable-density wind tunnel. Finally, Bacon & Reid (1924) have dropped some large spheres in free air and the resultant values of $C_D$ are shown in figure 5. It is assumed that these support-free data are representative of the value of $C_D$ in the supercritical regime.

On the basis of the foregoing discussion it seems reasonable to conclude that for $Re_\infty > 10^2$ the drag on a sphere in low turbulence flow is defined by the data obtained by Bailey & Hiatt (1971), Shakespear (1914), Vlajinac & Covert (1972) and Bacon & Reid (1924). A revised 'standard drag' curve based on these values is shown compared with the earlier 'standard drag' curve in figure 6.

## 5. Conclusions

The present study has shown that many of the measurements of subsonic sphere drag obtained to date have been affected by the methods used to obtain them. For example, it has been shown for $Re_\infty > 10^2$ that (i) translational oscillation of the sphere can cause an increase in the drag, (ii) flow turbulence in the medium through which the sphere is falling results in a decrease in drag, (iii) in some of the free-fall measurements the fall distance has been insufficient for the terminal velocity to have been achieved (consequently, the sphere drag coefficient has been over-estimated) and (iv) for supercritical flow the

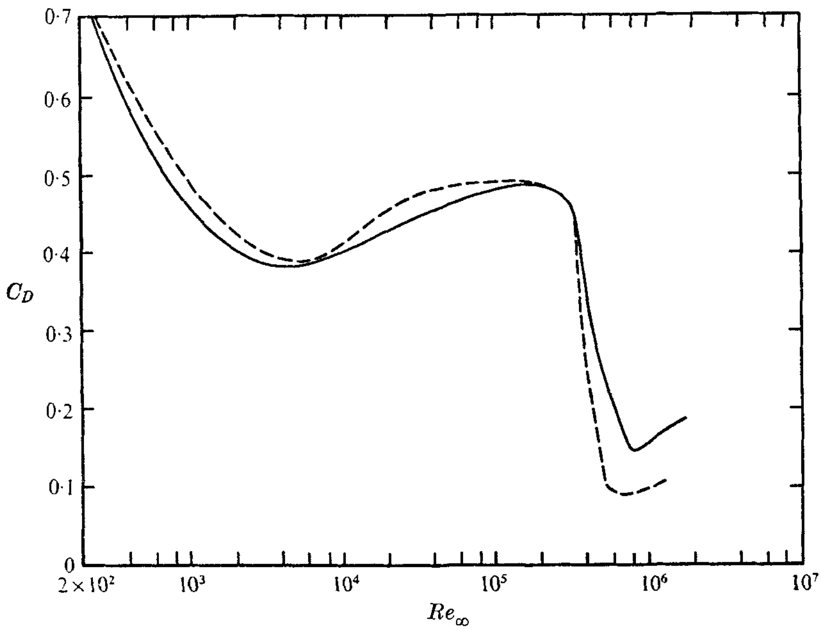


FIGURE 6. Sphere drag coefficient as a function of Reynolds number. ——, revised standard drag curve; — — — —, standard drag curve, Hoerner (1958).

absolute drag value is affected by flow turbulence and the model mounting technique.

When the above factors have been taken into consideration it has been shown for subcritical flow that most of the earlier data are in reasonable agreement with the results of the extensive aeroballistic-range study of Bailey & Hiatt (1971). For supercritical flow the existing measurements are characterized by considerable spread and the suggested value of $C_D$ is based on one set of free-fall data. To determine the value of $C_D$ in supercritical low turbulence flow it would be desirable to make more free-fall measurements.

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